

Rotational Dynamics

Object: To study rotational dynamics. This will be accomplished by determining the moment of inertia of a ring.

Apparatus: Rod stand, Rotary Motion Sensor, Disk, Ring, mass hanger and masses, string, balance, caliper, ruler, interface device, computer, and *DataStudio* software.

Foreword

Recall the statement of Newton's Second Law from **Experiment 4M**:

"The net force on an object is equal to the mass of the object times its acceleration."

Expressing this mathematically gives:

$$(1) \quad F_{\text{net}} = Ma$$

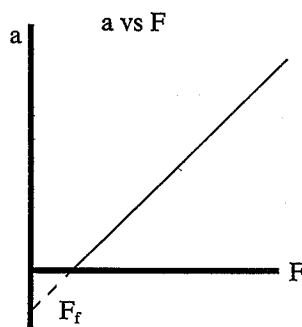
Now let's agree to write the net force F_{net} as the accelerating force, F , minus the force of friction, F_f . A mathematical expression of this statement is:

$$(2) \quad F_{\text{net}} = F - F_f$$

where F is the force tending to accelerate the mass M , and F_f is the force of friction tending to decelerate the mass M . Combining equations (1) and (2) and solving for the acceleration, a :

$$(3) \quad a = \frac{1}{M}F - \frac{1}{M}F_f$$

If the mass, M , is held constant, the accelerating force, F , varied, and the resulting acceleration determined, **Figure 1** shows a plot of the acceleration, a , versus the accelerating force, F .



Equation of the line

$$a = \frac{1}{M}F - \frac{1}{M}F_f$$

Slope = $1/M$

y-intercept = F_f/M

When $a = 0$, $F = F_f$

Figure 1

Recall that the equation for a straight line is:

$$(4) \quad y = mx + b$$

where y and x stand for the physical quantities plotted on the axes, m represents the physical slope, and b represents the intercept of the y -axis. When equation (3) is compared to equation (4), it may be concluded that the slope of the a vs. F plot is the reciprocal of the mass, M , being accelerated, and F_f/M has the value of the intercept of the ordinate. So, knowing M and the value of the intercept, one may determine the force of friction, F_f . Also, the force of friction, F_f , may be read directly from the x -axis for the special case where $a = 0$.

The physical quantities involved in the discussion above are force, mass, and acceleration. These physical quantities have the following rotational analogues.

Linear Physical Quantity	Symbol	Rotational Analogue	Symbol	Relation between linear quantity and its analogue
Force	F	Torque	τ	$\tau = rF$
Linear Acceleration	a	Angular Acceleration	α	$\alpha = a/r$
Mass	M	Moment of Inertia	I	$I = \sum_i m_i r_i^2$

Using these rotational analogues for the physical quantities of force, mass, and acceleration, one may obtain the statement of Newton's Second Law for rotational motion:

"The net torque on an object is equal to the moment of inertia of that object times its angular acceleration."

A mathematical expression of this statement is:

$$(5) \quad \tau_{\text{net}} = I\alpha .$$

In this experiment, the net torque is:

$$(6) \quad \tau_{\text{net}} = \tau - \tau_f$$

where τ is the torque tending to accelerate the object with moment of inertia, I , and τ_f is the torque of friction tending to decelerate the object.

Combining equations (5) and (6) and solving for the angular acceleration, α , one obtains

$$(7) \quad \alpha = \frac{1}{I}\tau - \frac{1}{I}\tau_f .$$

Equation (7) is the rotational analogue to equation (3) and the analogue plot α vs. τ is:

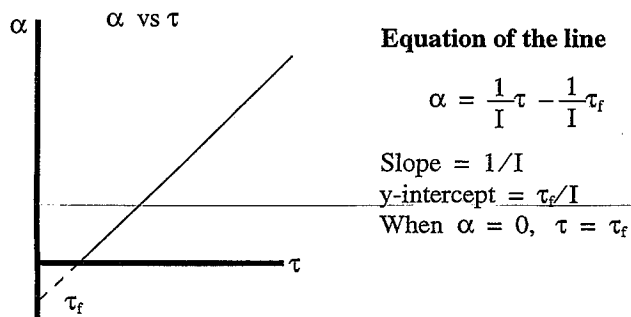


Figure 2

When (14) is compared to the equation for a straight line [see eq. (4)], one may conclude that the slope of the angular acceleration versus accelerating torque plot is the reciprocal of the moment of inertia of the object being accelerated. Also, the torque of friction may be obtained from the ordinate intercept of read directly from the x-axis for the special case of $a = 0$.

In order to create this plot to find the moment of inertia by a dynamical method, we need to investigate how to find the information to plot on the two axes.

The angular acceleration will be provided by the rotary motion sensor and calculated by the computer.

The torque will require a bit more work. First, we should remind ourselves of the definition of Torque.

(8)
$$\tau = r \leftrightarrow F$$

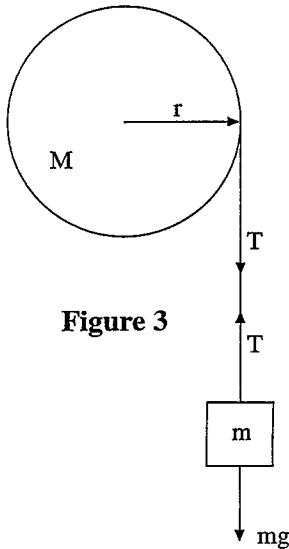


Figure 3

When we examine **Figure 3**, we can see that when a smaller mass, m , is dropped to accelerate a system of mass, M , and radius, r , the force causing the disk to rotate is the Tension, T . Also, we see that this tension is acting at a distance, r (the radius) away from the pivot point. Therefore, equation (8) becomes:

(9)
$$\tau = T * r$$

The radius can be easily measured. However, the tension will be determined by the smaller mass causing the system to rotate.

If we apply Newton's Second Law to the small mass m , (calling the downward direction of motion the positive direction) we get

(10)
$$F = ma$$

(11)
$$mg - T = ma$$

If we re-arrange this equation, we can solve for the Tension,

(12)
$$T = m(g - a)$$

When we substitute this expression for the Tension into equation (9) for the torque, we get:

(13)
$$\tau = mr(g - a)$$

Later, we will put this equation into the calculator on the computer so that they computer will calculate the torque for us as we vary the mass, m , and the computer measures the acceleration.

Since we have no way of rotating the ring by itself and measuring the torque and angular acceleration, we must apply an indirect method with what we do have. We can determine the moment of inertia, I_{disk} of the silver disk and everything inside the rotary motion sensor, then we can determine the moment of inertia, I_{sys} , for the ring and disk system. Subtracting these two quantities will give us the moment of inertia for the ring, I_{ring} .

(14)
$$I_{ring} = I_{system} - I_{disk}$$

As a method of comparison, we would like to determine the moment of inertia of the disk by a simple analytical method so that we have something with which to compare our value. This is done simply by finding a reference book which will give us an expression for the moment of inertia of a "thick ring" being rotated about its center. Such an expression can be found as:

(15)
$$I = \frac{1}{2} M [(R_1)^2 + (R_2)^2]$$

Where R_1 and R_2 are the inner and outer radii of the ring, and M is the mass of the ring as seen in **Figure 4**.

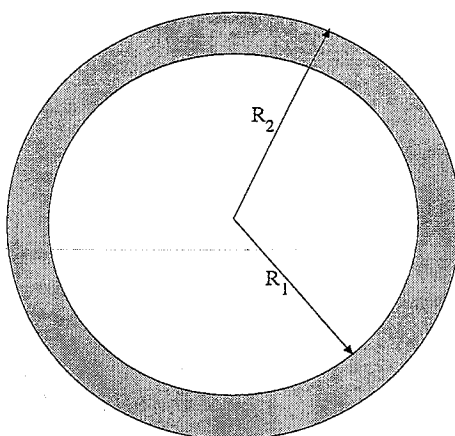


Figure 4

Procedure


Part I. Dynamical determination of moment of inertia of Ring and Disk

HARDWARE SETUP

1. Make sure interface box is turned on and connected to the computer, and make sure that the monitor, keyboard, and mouse are also connected to the computer.
2. Plug the YELLOW plug from the rotary motion sensor into Digital Channel 1 of the interface device, and plug the BLACK plug into Digital Channel 2.
3. Assemble the Rod Stand if necessary and mount the rotary motion sensor on the rod stand such that the disk rotates horizontal to the floor.
4. Attach the Super Pulley to the rotary motion sensor so that the string will easily run through the pulley when it is being unwound (the pulley will be at an angle).
5. Place the ring on top of the disk, and make sure it is properly locked into place (the pegs should fit into the small holes on top of the disk).

SOFTWARE SETUP

6. Open the *DataStudio* software by double clicking on the icon. If asked what action to take at this time, respond by clicking on the "Create Experiment" icon.
7. Tell the software you have plugged a **Rotary Motion Sensor** into Digital Channel 1 by dragging the icon for the rotary motion sensor from the sensor list to digital channel 1.
8. Double click the rotary motion sensor icon that is plugged into the Interface to bring up the Sensor Properties window.
 - a. Click on the **Measurement** tab to display the measurement list
 - b. Select Acceleration in (m/s/s) and Angular Acceleration in (rad/s/s) and DeSelect Angular Position
 - c. Click "OK" to close the window and save your changes.

9. We need to be able to inform the software of the different masses we will be using to increase the Torque throughout the experiment. To do this:
 - a. Bring up the Sampling Options window by clicking on the  button at the top of the Experiment Setup window.
 - b. Put a check in the box by the text, "**Keep data values...**" and the other 2 boxes underneath should become active with checks already in them.
 - c. Change the "Name" area of the Keyboard Data from "Keyboard 1" to "Mass" and enter the appropriate unit for mass in the "Units" area.
 - d. Click "OK" to exit and save your changes.

10. We will now use this information along with the information gathered from the rotary motion sensor to calculate the torque for each run. To set up the calculation:
 - a. Click on the calculator icon, labeled "Calculate" at the top of the window.
 - b. Enter the equation exactly as it is stated in Equation (13), (*you can change "y" to Torque*) and be sure to put multiplication signs between values that are multiplied. $Torque = m * r * (g - a)$
 - c. Then, click "Accept." The bottom of the window should expand and ask you to define each of your variables. Remember that "m" and "a" are Data Measurements and "r" and "g" are constants.
 - d. Define each variable appropriately by clicking on the icon to the left of each request and selecting the appropriate item from the drop down menu. You will need to know that the radius of the large disk that you are winding the string around is ($r = 0.02385\text{m}$)
 - e. Once all the variables are defined, you may close the calculator window.

11. In order to determine the moment of inertia, we need to graph angular acceleration vs. Torque. So, click and drag the "Angular Acceleration" data to the Graph icon. This tells the computer we want the angular acceleration, α , on the y-axis.

12. When the graph appears, click and drag the Torque calculation to the X-axis. Be sure there is a dashed box around the X-axis before releasing. Otherwise, you are probably not in the right location on the graph. You should now have a graph of angular acceleration vs. torque.

COLLECT DATA

13. Hang 10grams (0.010kg) *{including the mass of the mass hanger}* from the end of the string and wind the string around the large pulley by turning the disk until the mass hanger is just below the super pulley.
14. Click **Start**.
15. Release the disk letting the mass fall and the disk spin, and **while the system is in motion**, click the **KEEP** button to record data. A popup window will appear asking you to enter the appropriate mass for the acceleration which was just measured.
16. Add 10 grams, wind the string again and repeat the procedure until you have collected five data points and your mass, m , is 50g.
17. Click the red square to stop once your data is collected.

ANALYZE DATA

18. Scale the graph by clicking on the AUTOSCALE button.
19. Click on the "FIT" button and using the pull down menu, apply a linear fit to the graph and record the slope on the data sheet.
20. Use the information to determine the moment of inertia for the ring and disk system, I_{sys} .

Part II. Dynamical determination of moment of inertia of disk

HARDWARE SETUP

1. Remove the ring from the disk.

COLLECT & ANALYZE DATA

2. Since the software is already setup from the last run, repeat steps 13 – 20 from Part I.
3. Be sure to record the appropriate slope and moment of inertia for I_{disk} .
4. Print the graph.

DETERMINE VALUE OF MOMENT OF INERTIA OF RING

5. Subtract the two values as shown in equation (14) to get the moment of inertia of the ring.

Part III. Analytical determination of moment of inertia of ring

1. Using the caliper and balance provided, determine the appropriate values to calculate the analytical value for the moment of inertia of the ring.
2. Calculate the analytical value for the moment of inertia of the ring.
3. Compare this value to the dynamical value for the moment of inertia of the ring by calculating the percent difference and record all information on the data sheet.

DATA AND CALCULATION SHEET

Part I. Disk and Ring system

Slope from α vs. τ Graph: _____

Value for I_{sys} _____

Part II. Disk without Ring

Slope from α vs. τ Graph: _____

Value for I_{disk} _____

Value for I_{ring} _____

Part III. Analytical Determination of I_{ring}

Mass: _____

R_1 : _____

R_2 : _____

Value for I_{ring} : _____

COMPARE ANALYTICAL TO DYNAMICAL METHOD:

% difference: _____